Shut up and calculate*

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I advocate an extreme "shut-up-and-calculate" approach to physics, where our external physical reality is assumed to be purely mathematical. This brief essay motivates this "it's all just equations" assumption and discusses its implications.

What is the meaning of life, the universe and everything? In the sci-fi spoof *The Hitchhiker's Guide to the Galaxy*, the answer was found to be 42; the hardest part turned out to be finding the real question. Indeed, although our inquisitive ancestors undoubtedly asked such big questions, their search for a "theory of everything" evolved as their knowledge grew. As the ancient Greeks replaced myth-based explanations with mechanistic models of the solar system, their emphasis shifted from asking "why" to asking "how".

Since then, the scope of our questioning has dwindled in some areas and mushroomed in others. Some questions were abandoned as naive or misguided, such as explaining the sizes of planetary orbits from first principles, which was popular during the Renaissance. The same may happen to currently trendy pursuits like predicting the amount of dark energy in the cosmos, if it turns out that the amount in our neighbourhood is a historical accident. Yet our ability to answer other questions has surpassed earlier generations' wildest expectations: Newton would have been amazed to know that we would one day measure the age of our universe to an accuracy of 1 per cent, and comprehend the microworld well enough to make an iPhone.

Mathematics has played a striking role in these successes. The idea that our universe is in some sense mathematical goes back at least to the Pythagoreans of ancient Greece, and has spawned centuries of discussion among physicists and philosophers. In the 17th century, Galileo famously stated that the universe is a "grand book" written in the language of mathematics. More recently, the physics Nobel laureate Eugene Wigner argued in the 1960s that "the unreasonable effectiveness of mathematics in the natural sciences" demanded an explanation.

Here, I will push this idea to its extreme and argue that our universe is not just described by mathematics — it is mathematics. While this hypothesis might sound rather abstract and far-fetched, it makes startling predictions about the structure of the universe that could be testable by observations. It should also be useful in narrowing down what an ultimate theory of everything can look like.

The foundation of my argument is the assumption that there exists an external physical reality independent of us humans. This is not too controversial: I would guess that the majority of physicists favour this long-standing idea, though it is still debated. Metaphysical solipsists reject it flat out, and supporters of the so-called Copenhagen interpretation of quantum mechanics may reject it on the grounds that there is no reality without observation (New Scientist, 23 June, p 30). Assuming an external reality exists, physics theories aim to describe how it works. Our most successful theories, such as general relativity and quantum mechanics, describe only parts of this reality: gravity, for instance, or the behaviour of subatomic particles. In contrast, the holy grail of theoretical physics is a theory of everything — a complete description of reality.

My personal quest for this theory begins with an extreme argument about what it is allowed to look like. If we assume that reality exists independently of humans, then for a description to be complete, it must also be well-defined according to non-human entities — aliens or supercomputers, say — that lack any understanding of human concepts. Put differently, such a description must be expressible in a form that is devoid of any human baggage like "particle", "observation" or other English words.

In contrast, all physics theories that I have been taught have two components: mathematical equations, and words that explain how the equations are connected to what we observe and intuitively understand. When we derive the consequences of a theory, we introduce new concepts — protons, molecules, stars — because they are convenient. It is important to remember, however, that it is we humans who create these concepts; in principle, everything could be calculated without this baggage. For example, a sufficiently powerful supercomputer could calculate how the state of the universe evolves over time without interpreting what is happening in human terms.

All of this raises the question: is it possible to find a description of external reality that involves no baggage? If so, such a description of objects in this external reality and the relations between them would have to be completely abstract, forcing any words or symbols to be mere labels with no preconceived meanings whatsoever. Instead, the only properties of these entities would be those embodied by the relations between them.

This is where mathematics comes in. To a modern logician, a mathematical structure is precisely this: a set of abstract entities with relations between them. Take the integers, for instance, or geometric objects like the do-

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decahedron, a favourite of the Pythagoreans. This is in stark contrast to the way most of us first perceive mathematics — either as a sadistic form of punishment, or as a bag of tricks for manipulating numbers. Like physics, mathematics has evolved to ask broader questions.

Modern mathematics is the formal study of structures that can be defined in a purely abstract way. Think of mathematical symbols as mere labels without intrinsic meaning. It doesn't matter whether you write "two plus two equals four", "2 + 2 = 4" or "dos mas dos igual a cuatro". The notation used to denote the entities and the relations is irrelevant; the only properties of integers are those embodied by the relations between them. That is, we don't invent mathematical structures — we discover them, and invent only the notation for describing them.

So here is the crux of my argument. If you believe in an external reality independent of humans, then you must also believe in what I call the mathematical universe hypothesis: that our physical reality is a mathematical structure. In other words, we all live in a gigantic mathematical object — one that is more elaborate than a dodecahedron, and probably also more complex than objects with intimidating names like Calabi-Yau manifolds, tensor bundles and Hilbert spaces, which appear in today's most advanced theories. Everything in our world is purely mathematical — including you.

If that is true, then the theory of everything must be purely abstract and mathematical. Although we do not yet know what the theory would look like, particle physics and cosmology have reached a point where all measurements ever made can be explained, at least in principle, with equations that fit on a few pages and involve merely 32 unexplained numerical constants (Physical Review D, vol 73, 023505). So it seems possible that the correct theory of everything could even turn out to be simple enough to describe with equations that fit on a T-shirt.

Before discussing whether the mathematical universe hypothesis is correct, however, there is a more urgent question: what does it actually mean? To understand this, it helps to distinguish between two ways of viewing our external physical reality. One is the outside overview of a physicist studying its mathematical structure, like a bird surveying a landscape from high above; the other is the inside view of an observer living in the world described by the structure, like a frog living in the landscape surveyed by the bird.

One issue in relating these two perspectives involves time. A mathematical structure is by definition an abstract, immutable entity existing outside of space and time. If the history of our universe were a movie, the structure would correspond not to a single frame but to the entire DVD. So from the bird's perspective, trajectories of objects moving in four-dimensional space-time resemble a tangle of spaghetti. Where the frog sees something moving with constant velocity, the bird sees a straight strand of uncooked spaghetti. Where the frog sees the moon orbit the Earth, the bird sees two intertwined spaghetti strands. To the frog, the world is described by Newton's laws of motion and gravitation. To the bird, the world is the geometry of the pasta.

A further subtlety in relating the two perspectives involves explaining how an observer could be purely mathematical. In this example, the frog itself must consist of a thick bundle of pasta whose highly complex structure corresponds to particles that store and process information in a way that gives rise to the familiar sensation of self-awareness.

Fine, so how do we test the mathematical universe hypothesis? For a start, it predicts that further mathematical regularities remain to be discovered in nature. Ever since Galileo promulgated the idea of a mathematical cosmos, there has been a steady progression of discoveries in that vein, including the standard model of particle physics, which captures striking mathematical order in the microcosm of elementary particles and the macrocosm of the early universe.

That's not all, however. The hypothesis also makes a more dramatic prediction: the existence of parallel universes. Many types of "multiverse" have been proposed over the years, and it is useful to classify them into a four-level hierarchy. The first three levels correspond to non-communicating parallel worlds within the same mathematical structure: level I simply means distant regions from which light has not yet had time to reach us; level II covers regions that are forever unreachable because of the cosmological inflation of intervening space; and level III, often called "many worlds", involves non-communicating parts of the so-called Hilbert space of quantum mechanics into which the universe can "split" during certain quantum events. Level IV refers to parallel worlds in distinct mathematical structures, which may have fundamentally different laws of physics.

Today's best estimates suggest that we need a huge amount of information, perhaps a Googol (10^{100}) bits, to fully describe our frog's view of the observable universe, down to the positions of every star and grain of sand. Most physicists hope for a theory of everything that is much simpler than this and can be specified in few enough bits to fit in a book, if not on a T-shirt. The mathematical universe hypothesis implies that such a simple theory must predict a multiverse. Why? Because this theory is by definition a complete description of reality: if it lacks enough bits to completely specify our universe, then it must instead describe all possible combinations of stars, sand grains and such — so that the extra bits that describe our universe simply encode which universe we are in, like a multiversal telephone number. In this way, describing a multiverse can be simpler than describing a single universe.

Pushed to its extreme, the mathematical universe hypothesis implies the level-IV multiverse, which includes all the other levels within it. If there is a particular mathematical structure that is our universe, and its properties correspond to our physical laws, then each mathematical structure with different properties is its own universe with different laws. Indeed, the level-IV multiverse is compulsory, since mathematical structures are not "created" and don't exist "somewhere" — they just exist. Stephen Hawking once asked, "What is it that breathes fire into the equations and makes a universe for them to describe?" In the case of the mathematical cosmos, there is no fire-breathing required, since the point is not that a mathematical structure describes a universe, but that it is a universe.

The existence of the level-IV multiverse also answers a confounding question emphasised by the physicist John Wheeler: even if we found equations that describe our universe perfectly, then why these particular equations, not others? The answer is that the other equations govern parallel universes, and that our universe has these particular equations because they are statistically likely, given the distribution of mathematical structures that can support observers like us.

It is crucial to ask whether parallel universes are within the purview of science, or are merely speculation. Parallel universes are not a theory in themselves, but rather a prediction made by certain theories. For a theory to be falsifiable, we need not be able to observe and test all its predictions, merely at least one of them. General relativity, for instance, has successfully predicted many things that we can observe, such as gravitational lensing, so we also take seriously its predictions for things we cannot, like the internal structure of black holes.

So here's a testable prediction of the mathematical universe hypothesis: if we exist in many parallel universes, then we should expect to find ourselves in a typical one. Suppose we succeed in computing the probability distribution for some quantity, say the dark energy density or the dimensionality of space, measured by a typical observer in the part of the multiverse where this quantity is defined. If we find that this distribution makes the value measured in our own universe highly atypical, it would rule out the multiverse, and hence the mathematical universe hypothesis. Although we are still far from understanding the requirements for life, we could start testing the multiverse prediction by assessing how typical our universe is as regards dark matter, dark energy and neutrinos, because these substances affect only better understood processes like galaxy formation. This prediction has passed the first of such tests, because the abundance of these substances has been measured to be rather typical of what you might measure from a random galaxy in a multiverse. However, more accurate calculations and measurements might still rule out such a multiverse.

Ultimately, why should we believe the mathematical universe hypothesis? Perhaps the most compelling objection is that it feels counter-intuitive and disturbing. I personally dismiss this as a failure to appreciate Darwinian evolution. Evolution endowed us with intuition only for those aspects of physics that had survival value for our distant ancestors, such as the parabolic trajectories of flying rocks. Darwin's theory thus makes the testable prediction that whenever we look beyond the human scale, our evolved intuition should break down.

We have repeatedly tested this prediction, and the results overwhelmingly support it: our intuition breaks down at high speeds, where time slows down; on small scales, where particles can be in two places at once; and at high temperatures, where colliding particles change identity. To me, an electron colliding with a positron and turning into a Z-boson feels about as intuitive as two colliding cars turning into a cruise ship. The point is that if we dismiss seemingly weird theories out of hand, we risk dismissing the correct theory of everything, whatever it may turn out to be.

If the mathematical universe hypothesis is true, then it is great news for science, allowing the possibility that an elegant unification of physics and mathematics will one day allow us to understand reality more deeply than most dreamed possible. Indeed, I think the mathematical cosmos with its multiverse is the best theory of everything that we could hope for, because it would mean that no aspect of reality is off-limits from our scientific quest to uncover regularities and make quantitative predictions.

However, it would also shift the ultimate question about the universe once again. We would abandon as misguided the question of which particular mathematical equations describe all of reality, and instead ask how to compute the frog's view of the universe — our observations — from the bird's view. That would determine whether we have uncovered the true structure of our universe, and help us figure out which corner of the mathematical cosmos is our home.

[1] M Tegmark 2007, "The Mathematical Universe", arXiv 0704.0646 [gr-qc], submitted to *Foundations of Physics*